E. APPENDIX - Notes and Errata

The notes listed below are included to bring attention to notation and other problems. The page number provides reference to the location of the problem or comment.

1. page TN-4

AM noise is especially important in the measurement of the (residual) phase noise added by amplifiers and other signal handling components. Often the driving source for the measurements is a frequency synthesizer that has phase and/or amplitude noise that is comparable or larger than the added noise of the component under test. Most measurement systems are configured so that the phase noise of the source cancels out to a large degree (at high Fourier frequencies decorrelation effects limit the cancellation). In such measurement systems, the AM to PM conversion factor and the AM noise of the source may then set the noise floor. Discussion of the effect of AM noise and AM to PM conversion factors on the accuracy and precision of phase noise measurements is found in "Residual Phase Noise Measurements of vhf, uhf, and Microwave Components" by G. K. Montress, T. E. Parker, and M. J. Loboda Proc. *43rd Annual Symposium on Frequency Control*, pp. 349-359 (1989) and in "Accuracy Model for Phase Noise Measurements by F. L. Walls, C. M. Felton, and A. J. D. Clements, *21st Annual Precision Time and Time Interval Meeting* (1989). The notation in these papers as well as that in other parts of the literature differs from that given below. Our notation below is drawn from a modest level of consensus among individuals responsible for setting standards. We expect that it will gradually be adopted within standards committees. The following comments are directed specifically at the specification of noise performance.

The total power spectral density in a signal can be approximated by expanding eq 12-5 of paper B.2 (by Stein) and extending it to include the spectral density of relative amplitude fluctuations, $S_a(f)$. The double-sideband density written in single-sideband form is given by

$$S_V(f) \simeq \frac{V_0^2}{2}\left[e^{-I(f_c)}\delta(f) + e^{-I(f)}S_\phi(f) + S_a(f)\right] \qquad 0 < f < \infty,$$

where

$$I(f) \simeq \int_f^\infty S_\phi(f)df.$$

I(f) is the integrated phase modulation due to the pedestal and $\delta(f)$ represents the carrier with frequency width $\pm f_c$. The effect of large $S_a(f)$ on power in the carrier has not, to our knowledge, been explored. The power spectral density of relative phase fluctuations, $S_\phi(f)$, is normalized to one rad^2/Hz and $S_a(f)$ is normalized to the carrier voltage, but the total power spectral density, $S_V(f)$ is not normalized and has the units of V^2/Hz. All of these are single-sided spectral densities. For most measurement purposes, we can disregard the carrier and find that, away from the carrier, the above expression simplifies to

$$S_V(f) \simeq \frac{V_0^2}{2}\left[S_\phi(f) + S_a(f)\right] \qquad for \qquad 0 \le f \le \infty.$$

The more general expression is important only very near the carrier and in certain types of frequency multiplication. The single-sideband, amplitude noise, normalized to the total signal power is given simply by $S_a(f)$ for $0 \leq f \leq \infty$. The measurement of added phase and amplitude noise for amplifiers and other signal handling components should specify the signal level since the AM noise level and the contribution due to AM-to-PM conversion depend on the signal level.

2. <u>page TN-6</u>
For a direct measurement, time accuracy only has meaning when the phase of the time-base oscillator of the frequency counter is known with respect to some time standard. Either it is phase locked or is calibrated with respect to that standard at the time of measurement. The phase of the time-base oscillator can then be measured with respect to the phase of the frequency standard being calibrated (accounting for cable delays, etc.). Except for the cycle ambiguity of the carrier, the phase of the frequency standard being measured can carry time information and have time accuracy. This technique is not common, but is very useful and eliminates divider noise that typically occurs in going from 5 or 10 Mhz to 1 pulse per second. Caution must be exercised to assure that the phase point measured in a sine wave is at a reproducible voltage and impedance so that the cycle ambiguity is an exact integer.

3. <u>page TN-35</u>
A second-order servo loop provides substantially enhanced performance. See, for example, F.L. Walls and S.R. Stein, "Servo techniques in oscillators and measurement systems," *NBS Tech. Note 692* (1976).

4. <u>page TN-35</u>
This error can be identified and corrected using the phase modulation scheme described in paper B.4 on page TN-136.

5. <u>page TN-36</u>
Low-noise DC amplifiers have been substantially improved since publication of this paper.

6. <u>pages TN-37, TN-91, TN-130, TN-174, TN-206 and TN-218</u>
The reader is reminded that the discussions of frequency-domain measurements assume incoherent noise processes. Often the phase noise spectrum of a signal will contain bright spectral features (spurious lines) other than the carrier. Frequency-domain measurements are often useful in identifying such features. But if these spurious lines are narrow compared to the measurement bandwidth, statistical measures such as $S_\phi(f)$ and $\mathcal{L}(f)$ are not appropriate. It is better to specify the phase deviation in terms of the rms value of the phase deviations, ϕ_{rms}(rms radians), without reference to bandwidth. This specification in rms radians can be related to the Allan variance (see note # 8 below).

7. <u>page TN-51</u>
Humidity is often an important environmental factor. See, for example, J.E. Gray, H.E. Machlan and D.W. Allan, "The Effect of humidity on commercial cesium beam atomic clocks," *42nd Annual Symp. on Frequency Control*, pp. 514-518 (1988) and F.L. Walls, "The

Influence of Pressure and Humidity on the Medium and Long-Term Frequency Stability of Quartz Oscillators," *42nd Annual Symp. on Frequency Control*, pp. 279-283 (1988).

8. page TN-74

For a bright line (one which is narrow compared with the measurement bandwidth), the solution of eq 12-27 simplifies to

$$\sigma_y(\tau) = \frac{\sqrt{2}\,\phi_{rms}}{\pi \nu_0 \tau}\sin^2(\pi f \tau),$$

where ϕ_{rms} is the rms value of the phase deviations. The above relationship may be useful where one is trying to determine the effect of a bright line in the time domain. If the bright line is the dominant factor, the plot of $\sigma_y(\tau)$ versus τ has strong $\sin^2(\pi f \tau)$ oscillations and it can be ambiguous. In that case, it is better to provide a specification in terms of ϕ_{rms} without reference to bandwidth. Statistical measures such as $\sigma_y(\tau)$ and $S_\phi(f)$ are not meant to be used to describe coherent signals. For further discussion see paper B.1, section 12.2 (page TN-51).

9. page TN-75

A set of brackets, [ ], are missing in eq 12-29. The equation should read

$$\mathrm{mod}\,\sigma_y^2(\tau) = \frac{1}{2\tau^2 N^2 (N-3n+1)} \sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{n+j-1}(x_{i+2n}-2x_{i+n}+x_i)\right]^2 .$$

10. page TN-119

The reference (Walls and DeMarchi, 1975) is listed as being on pages 310-317. The page numbers should be 210-217.

11. page TN-121

Most of the literature uses the expression $[V_0 + \epsilon(t)]$ instead of V_0 as in eq (2). V_0 is the peak voltage amplitude and $\epsilon(t)$ is the voltage deviation of the amplitude from nominal.

12. page TN-122

In eq (5), most of the literature uses $x_1(t)$ instead of $\epsilon(t)$. $\epsilon(t)$ is usually the voltage deviation as described in note 11 above.

13. page TN-123

There is an error in the caption for figure 7. The last portion of that caption should read: "where f is Fourier frequency (($\omega = 2\pi f$) and $S_y(f) = \omega^2 S_x(f)$)."

14. page TN-123

In the right-hand column, last paragraph, 5th line, there is an extraneous minus sign. The quantity $\overline{y}_i^{-\tau_0}$ should read $\overline{y}_i^{\tau_0}$. Also, note that the use of supercript τ and τ_0 with $\overline{y}$ is not consistent with the new IEEE standard definitions (see paper C.1, page TN-139).

15. page TN-124

In eq 9 the subscript, k - n, should read k + n. That is, the equation should read

$$\bar{y}_k^{\tau} = \frac{1}{n}\sum_{i=k}^{k+n-1} \bar{y}_i^{\tau_0} = \frac{x_{k+n} - x_k}{\tau}.$$

16. page TN-125

The quantity $\bar{\sigma}_y^2(\tau)$ is now commonly known as mod$\sigma_y^2(\tau)$. This latter form has been recently adopted by IEEE as the standard terminology.

17. page TN-139

IEEE Std 1139-1988, *IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology*, is an almost exact replica of this paper (D.1). The paper was published during the latter period of the development of the standard. The only substantial difference is that, wherever it occurs, the word "departure" in the paper is replaced in the IEEE standard by "deviation."

18. page TN-146

This widely cited paper provided the *de facto* standards for terminology and oscillator characterization until the recent adoption of the IEEE standard presented in paper C.1 (page TN-139). For terminology, the latest IEEE standard should always take precedence. This paper (C.2) is fairly consistent with the IEEE standard. One exception is that, in this paper, N denotes the number of frequency measurements. The symbol M in the IEEE standard is the same as N in this paper. In the standard, the equation relating M and N is M = N - 1.

19. page TN-151

Equation (23) can be substantially simplified as shown, for example, by Stein (page TN-74, eq (12-27)), which is

$$\sigma_y^2(\tau) = \frac{2}{(\pi\nu_0\tau)^2}\int_0^{\infty} S_\phi(f)\sin^4(\pi f\tau)\,df.$$

20. page TN-154

In eq 36 the T outside the brackets should be a τ. The equation should read

$$\hat{x}(t_0+\tau) = x(t_0) + \tau\left[\frac{x(t_0) - x(t_0 - T)}{T}\right].$$

21. page TN-160

The 2 expressions listed as eq (95) are in error. They should read

$$\sim h_{-1}\tau^2[3 + 2\ln r - 1/(6r^2)] \qquad r > 1$$

$$\sim h_{-1}T^2[3 - 2\ln r] \qquad r < 1.$$

22. <u>page TN-160</u>

In eqs (101), (102), (103), (104), and (105), a Greek γ was mistakenly replaced by the number 2. In each of these equations the quantity $[2 + \ln(2\pi f_h \tau)]$ should be replaced by the quantity $[\gamma + \ln(2\pi f_h \tau)]$. γ, Euler's constant, has the value 0.5772156649........

23. <u>page TN-162</u>

This paper is included in this collection because it presents the internationally accepted terminology and definitions. There are no substantial inconsistencies with the new IEEE standard (paper C.1, page TN-139), but the latest IEEE standard should is considered to be the most up-to-date authority. A new version of this international report should be issued by the CCIR in 1990.

24. <u>page TN-171</u>

The definitions for symbols used in this paper are fairly consistent with those adopted in the recent IEEE standard (C.1). One exception is that, in this paper, N denotes the number of frequency measurements. The symbol M in the IEEE standard is the same as N in this paper. Another is that, while this paper uses μ as the exponent of τ in describing the power-law noise processes, the paper adopts the opposite sign convention for μ. v(t) is used where many other papers use V(t) for instantaneous voltage. Some confusion is generated when this small v is typeset in the equations to look almost identical to the Greek ν, a symbol which is used exclusively to represent frequency. For example, In equation (1) the left-hand quantity is voltage, whereas the $\nu(t)$ and ν_0 in equation (4) are clearly frequencies. Finally, the authors of this paper, in equation (2), define $\epsilon(t)$ as the normalized amplitude fluctuations, a very sound choice, but the reader should note that most other papers have not normalized it.

25. <u>page TN-175</u>

For consistency with figure 12 and the text, $\nu(t)$, the left-hand member of eq (11) should probably be u(t).

26. <u>page TN-177</u>

Walls, Percival and Irelan (D.4) have recently addressed the more accurate specification of the quantity p in eq (12).

27. <u>page TN-179</u>

It is important to note that the expressions in Table 2 in this paper are derived assuming use of a single-pole filter. The calculations can also be done using an ideal (infinitely sharp) filter. The solutions in these two limits are useful because they define the range of practical values (using n-pole filters) for the expressions. Table I in this section is an expansion of Table 2 of Lesage and Audoin providing both the single-pole results as well as the results for an infinitely sharp filter. There are discrepencies in several of the coefficients between terms in Table 2 in the paper and those in Table I on the next page.

28. <u>page TN-180</u>

Barnes and Allan (paper D.8) have recently completed further analysis of the effect of dead time on measurements.

Table I. Asymptotic forms of $\sigma_y^2(\tau)$ for various power-law types and two filter types. Note: $\omega_h/2\pi = f_h$ is the measurement system bandwidth, often called the high-frequency cutoff. $\ln \equiv \log_e$.

Name of Noise	**α**	**$S_y(f)$**	**$\sigma_y^2(\tau)$**			
			$\omega_h\tau >> 1$ Infinite Sharp Filter	$\omega_h\tau >> 1$ Single Pole Filter	$\omega_h\tau << 1$ Infinite Sharp Filter	$\omega_h\tau << 1$ Single Pole Filter
White Phase	2	$h_2 f^2$	$\frac{3f_h h_2}{(2\pi)^2\tau^2}$	$\frac{3f_h h_2}{(2\pi)^2\tau^2}$	$\frac{2\pi^2 f_h^5 \tau^2 h_2}{5}$	$\frac{f_h^2 h_2}{2\tau}$
Flicker Phase	1	$h_1 f$	$\frac{(1.038 + 3\ln(\omega_h\tau))h_1}{(2\pi)^2\tau^2}$	$\frac{(3\ln(\omega_h\tau))h_1}{(2\pi)^2\tau^2}$	$\frac{\pi^2 f_h^4 \tau^2 h_1}{2}$	$2f_h^2(\ln(2))h_1$
White Frequency	0	h_o	$\frac{h_o}{2\tau}$	$\frac{h_o}{2\tau}$	$\frac{2\pi^2 f_h^3 \tau^2 h_0}{3}$	$\frac{2\pi^2 f_h^2 \tau h_0}{3}$
Flicker Frequency	-1	$h_{-1} f^{-1}$	$2(\ln(2))h_{-1}$	$2(\ln(2))h_{-1}$	$\pi^2 f_h^2 \tau^2 h_{-1}$	$8\pi^2 f_h^2 \tau^2 h_{-1}$
Random-Walk Frequency	-2	$h_{-2} f^{-2}$	$\frac{2\pi^2\tau h_{-2}}{3}$	$\frac{2\pi^2\tau h_{-2}}{3}$	$2\pi^2 f_h \tau^2 h_{-2}$	$2\pi^2 f_h \tau^2 h_{-2}$

29. page TN-180
If the ratio of T/τ is constant and greater than 1 (the usual case), the problem described is eliminated. However, in taking data for a plot of $\sigma_y(\tau)$ versus τ, it is difficult to achieve this in the hardware and not possible to do it with software processing alone. For further discussion see paper D.8 (page TN-296).

30. page TN-197
The most recent definitions and concepts for spectral density are given in a new IEEE standard (paper C.1 on page TN-138). This new standard should be consulted as the latest authority on definitions and terminology.

31. page TN-198
The newly accepted definition of $\mathcal{L}(f)$ is given in paper C.1. This new definition, $\mathcal{L}(f) \equiv \frac{1}{2}S_\phi(f)$, was always valid for Fourier frequencies far from the carrier. It has now been extended to cover all frequencies.

32. page TN-217
Equation (73) should read 20 log(final frequency/original frequency).

33. page TN-239
On page TN-198 the authors refer to a paper by Glaze (1970). The reference, apparently lost in printing, is: Glaze, D.J. (1970). "Improvements in Atomic Beam Frequency Standards at the National Bureau of Standards," *IEEE Trans. Instrum. Meas.* **IM-19**(3), 156-160.

34. page TN-257
There are two errors in Table 2. Under R(n) the first entry should be 1/n rather than 1. The second item in the same column is not single valued (1), but takes on different values for different measurement bandwidths. The reader is referred to section A.6 (page TN-9) for a discussion of this topic.

35. pages TN-261 and TN-262
Subsequent work on $\mathrm{mod}\sigma_y^2(\tau)$ and R(n) is reported in section A.6 (page TN-9) of this report. There are some differences between the results in A.6 and the ones reported in Tables I and II and Figure 4 in this paper.

36. page TN-264
To be consistent with other papers in the literature, $\phi(t)$ in eq (1) should probably be written as $x_1(t)$.

37. page TN-268
The term $\epsilon(t)$ in eq (3) is normally used to represent the amplitude fluctuations in the output voltage of an oscillator. This term might be better designated $Y_1(t)$.